

Monopole-antimonopole condensation in the interpolating Georgi-Glashow model

Chulwoo Jung ^{a *}

^aDepartment of Physics, Columbia University, New York, NY 10027 U.S.A.

We study the three dimensional Georgi-Glashow model (which interpolates smoothly between pure U(1) and SU(2) limits) using a constrained cooling which preserves 't Hooft-Polyakov monopoles. We find that the monopole-antimonopole condensation gives an area law for the Wilson loops. The monopole contribution to the string tension is close to the Monte Carlo value in the intermediate region.

In spite of considerable evidence for confinement at weak coupling from numerical simulations, an analytic understanding of the long-range properties of non-Abelian gauge theories has been quite limited. In this note, we investigate the role of the 't Hooft-Polyakov monopoles in the non-perturbative physics of the 3 dimensional Georgi-Glashow model(GG_3). It was shown by Polyakov[1] that the condensation of these monopoles is responsible for confinement in the region where monopoles are far apart and the distribution of the monopoles is determined by Coulomb interaction between monopoles. Cooling (systematic reduction of the action)[2], when applied to the Monte Carlo lattices in the intermediate region of GG_3 , yields collections of (anti)monopoles which are stable solutions on the lattice. However, the distribution of monopoles in these configurations may have little bearing on the monopole distribution in the original configurations since cooling can change the monopole distribution.

In this work, the role of the 't Hooft-Polyakov monopoles present in the Monte Carlo(MC) lattices is investigated using a constrained cooling procedure we refer as monopole preserving cooling (MP cooling in short) which preserves the number and the location of the monopoles while cooling. A brief description of MP cooling and the comparison between MP cooling and the uncon-

strained cooling('cooling' in short) is given in section 1. We discuss the behavior of the Creutz ratios (both fundamental and adjoint) of MP cooled lattices in section 2.

1. MP cooling

For the numerical simulation, we used a discretized version of the Georgi-Glashow Lagrangian with the radially frozen approximation. ($\vec{\Phi}_{\mathbf{n}}$ is a real 3-vector, $\vec{\Phi}_{\mathbf{n}} \cdot \vec{\Phi}_{\mathbf{n}} = 1$)

$$S(\beta_g, \beta_h) = \beta_g S_g + \beta_h S_h \equiv \beta_g \sum_P (1 - \frac{1}{2} \text{tr}(U_P)) + \beta_h \sum_{\mathbf{n}\mu} (1 - \frac{1}{2} \text{tr}(\vec{\Phi}_{\mathbf{n}} \cdot \vec{\sigma} U_{\mathbf{n}\mu} \vec{\Phi}_{\mathbf{n}+\hat{\mu}} \cdot \vec{\sigma} U_{\mathbf{n}\mu}^\dagger))$$

This model describes a U(1) theory as $\beta_h \rightarrow \infty$ and the pure SU(2) theory as $\beta_h \rightarrow 0$.

MP cooling on the MC configurations is implemented as follows. After gauge transforming the Higgs fields to $\vec{\Phi} = \hat{z}$, the projected U(1) field is given by $\theta_{\mathbf{n}\mu} = \arg((U_{\mathbf{n}\mu})_{11})$ ($-\pi < \theta < \pi$). Now the monopoles are located on the dual lattice points by applying a standard procedure devised by DeGrand & Toussaint[3] on $\theta_{\mathbf{n}\mu}$. A new gauge link which lowers the action is accepted only if the monopole numbers for the 4 cubes surrounding the gauge link remains the same.

Table 1 shows the action density and the monopole density for 24^3 lattices. We used the couplings (β_g, β_h) obtained by Duncan and Mawhinney, which keep the string tension qualitatively the same[2]. At $\beta_g = 3.0, \beta_h = 3.0$, the

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β_g	3.0	6.5	7.6	9.4	9.9
β_h	3.0	1.0	0.87	0.5	0.1
S_g /plaquette	0.26341(10)	0.15472(3)	0.13422(3)	0.1094(5)	0.1038(5)
S_h /link	0.22705(6)	0.5068(3)	0.5962(3)	0.8200(33)	0.9667(28)
MP S_g /plaquette	0.00174(7)	0.00144(2)	0.002086(7)	0.00114(3)	0.00012(2)
MP S_h /link	0.000196(8)	0.0434(4)	0.1181(3)	0.4003(59)	0.8086(33)
Monopole density	0.01494(4)	0.0268(2)	0.0646(2)	0.1984(2)	0.2776(2)
MC String tension	0.0288(19)	0.0206(19)	0.0323(9)	0.0271(12)	0.0241(5)
MP Creutz ratio	0.0216(21)	0.0119(20)	0.0268(18)	0.0157(7)	

Table 1

The summary of measured quantities. All the simulations are done on a 24^3 lattice. The quoted value of the MC string tension is the constant fit to the Creutz ratio $C(R, R)$ for $R = 8$ to $R = 12$. MP Creutz ratio is $C(R, R)$ at $R = 12$. MP Creutz ratio may not be close to the asymptotic value due to the slow convergence as you see in Fig. 2.

MP cooled configuration is essentially $U(1)(S_h \sim 0)$ since the nonabelian cores of the monopoles are small. As we approach the $SU(2)$ limit, the monopole density increases since the action of individual monopoles gets smaller.

Table 2 shows the evolution of the monopole number and the action density of a single lattice near the $U(1)$ limit ($\beta_g = 3.0, \beta_h = 3.0$) and near the $SU(2)$ limit ($\beta_g = 9.9, \beta_h = 0.1$) under cooling and MP cooling. Cooling induces the annihilation of close monopole-antimonopole pairs, resulting in a lower action than MP cooling.

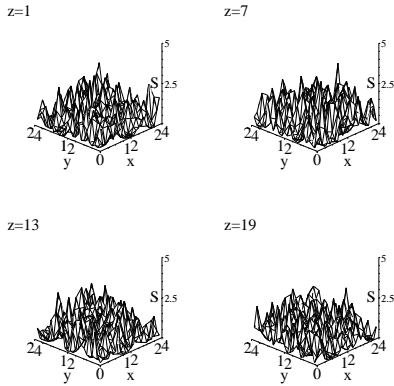


Figure 1. Action density plot of a MP cooled lattice for $\beta_g = 9.4$ $\beta_h = 0.5$.

Fig. 1 shows the action density plot of a MP cooled lattice near the $SU(2)$ limit ($\beta_g = 9.4, \beta_h = 0.5$). In contrast to the smooth action density we see for cooled lattices, MP cooled lat-

$\beta_g = 3.0$ $\beta_h = 3.0$					
	cooling			MP cooling	
N	mpl	S_g	S_h	S_g	S_h
0	24	0.2629	0.2284	0.2629	0.2284
2	16	8.82e-2	9.62e-2	8.82e-2	9.61e-2
10	12	1.58e-3	1.69e-4	2.08e-2	4.83e-4
50	12	1.26e-3	1.1e-10	1.75e-3	3.41e-4

$\beta_g = 9.9$ $\beta_h = 0.1$					
	cooling			MP cooling	
N	mpl	S_g	S_h	S_g	S_h
0	3872	0.1041	0.9696	0.1041	0.9696
10	3446	3.35e-4	0.9338	5.41e-4	0.9389
50	2534	9.95e-5	0.7728	1.65e-4	0.8111
100	1606	6.17e-5	0.5893	1.24e-4	0.6979
200	616	4.03e-5	0.3057	1.04e-4	0.5977
300	210	3.63e-5	0.1434	9.50e-5	0.5663

Table 2

The evolution of the monopole number and the action density under cooling and MP cooling. For MP cooling, the monopole number is fixed. cooling. N is the number of cooling steps.

tices show many small but sharp peaks and seems to lack any global structure which might develop into the smooth peaks when we approach the $SU(2)$ limit.

2. Creutz ratios

Fig. 2 shows the behavior of the fundamental Creutz ratios for the MC and the MP cooled lattices in the intermediate region ($\beta_g = 7.6, \beta_h =$

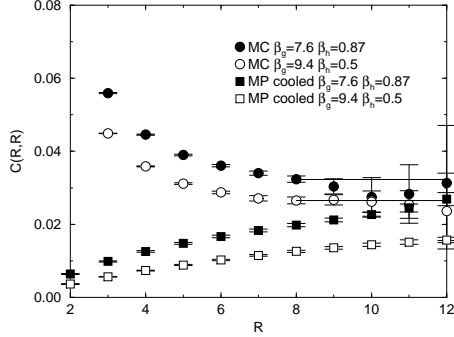


Figure 2. Creutz ratios for the MC and MP cooled lattices. Straight lines indicates the fitted value of the string tension for the MC lattices.

0.87) and near the SU(2) limit ($\beta_g = 9.4, \beta_h = 0.5$). While the presence of the Coulomb interaction makes the fundamental Creutz ratio for the MC lattices approach the asymptotic value from above, the Creutz ratio for MP cooled lattices approaches from below, similar to the behavior of the Creutz ratio of the monopole contribution in the Villain U(1) model[2]. From near the U(1) limit to the intermediate region, the MP Creutz ratios approach the MC Creutz ratios at a large R and a plateau begins to emerge. As we move closer to the SU(2) limit where the nonabelian cores of the monopoles begin to overlap, the area law for the Wilson loops persists although the Creutz ratio increases slower. It is not clear if the MP Creutz ratio converges to a smaller value or the lattice size is too small compared to the length scale at which it converges.

Fig. 3 shows the ratios of the fundamental and the adjoint Creutz ratio for the MC and the MP cooled lattices. The ratio is asymptotically 0 for a U(1) theory and is $8/3$ for pure SU(2) gauge theory until the onset of the color screening (Casimir scaling)[4]. In the intermediate region, the ratios for both the MC and MP cooled lattices show the transition between two values. Although the individual (fundamental, adjoint) Creutz ratios are quite different, the ratios of two Creutz ratios for MP cooled lattices show the same features as the MC lattices.

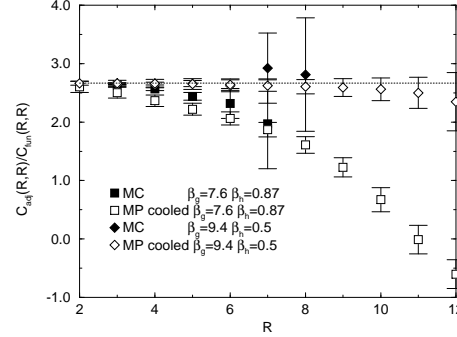


Figure 3. Ratios of the fundamental and the adjoint Creutz ratio of the MC and the MP cooled lattice. The dotted line indicates the Casimir ratio($8/3$). The ratio is zero at a large R for both the MC and the MP cooled lattices at $\beta_g = 3.0, \beta_h = 3.0$.

3. Discussion

The 't Hooft-Polyakov monopole condensation in GG_3 gives an area law for the Wilson loops although the string tension appears to be less than the MC value, especially near the SU(2) limit. The non-vanishing Higgs action and the Casimir scaling of the MP cooled lattices near the SU(2) limit suggests the interactions between these monopoles are no longer abelian. More simulation with a larger volume is needed to see whether the smaller Creutz ratios for MP cooled lattices are due simply to the small volume or a change in the confinement mechanism in this region.

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